BUFFET RESPONSES OF A VERTICAL TAIL IN VORTEX BREAKDOWN FLOWS

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ABSTRACT

The tail buffet problem is a multi-disciplinary problem which is solved using three sets of equations. The first set is the unsteady, compressible, full Navier-Stokes equations which are used for obtaining the flow-field vector and the aero-dynamic loads. The second set is the aeroelastic equations which are used for obtaining the bending and torsional deflections of the tail. The third set is the grid-displacement equations which are used for updating the grid coordinates due to the tail deflections. For the computational applications a sharp-edged cropped delta wing of aspect ratio 1.5 and a swept back vertical tail of aspect ratio 1.4 placed in the plane of geometric symmetry behind the wing are considered. The configuration is pitched at angles of attack of 20° and 28° which produce vortex breakdown flow of the delta wing primary vortices for the transonic regime at a Mach number of 0.85. The results show the effects of the angle of attack and vortex breakdown on the uncoupled bending-torsional responses.

INTRODUCTION

The ability of modern fighter aircraft to fly and maneuver at high angles of attack and at high loading conditions is of prime importance. This capability is achieved, for example in the F/A-18 fighter, through the combination of the leading-edge extension (LEX) with a delta wing and the use of vertical tails. The LEX maintains lift at high angles of attack by generating a pair of vortices that trail aft over the top of the aircraft. The vortex entrains air over the vertical tails to maintain stability of the aircraft. This combination of LEX, delta wing and vertical tails leads to the aircraft excellent high angle of attack performance. However, at some flight conditions, the vortices emanating from the highly-swept LEX of the delta wing breakdown before reaching the vertical tails which get bathed in a wake of unsteady highly-turbulent, swirling flow. The vortex-breakdown flow produces unsteady, unbalanced loads on the vertical tails which in turn produce severe buffet on the tails and has led to their premature fatigue failure.

Experimental investigation of the vertical tail buffet of the F/A-18 models have been conducted by several investigators such as Sellers at al1., Erickson at al2., Wentz3 and Lee and Brown4. These experiments showed that the vortex produced by the LEX of the wing breaks down ahead of the vertical tails at angles of attack of 25° and higher and the breakdown flow produced unsteady loads on the vertical tails. Rao, Puram and Shah5 proposed two aerodynamic concepts for alleviating high-alpha tail buffet characteristics of the twin tail fighter configurations. Cole, Moss and Doggett6 tested a rigid, 1/6 size, full-span model of an F-18 airplane that was fitted with flexible vertical tails of two different stiffness. Vertical-tail buffet response results were obtained over the range of angle of attack from -10° to +40°, and over the range of Mach numbers from 0.3 to 0.95. Their results indicated that the buffet response occurs in the first bending mode, increases with increasing dynamic pressure and is larger at $M = 0.3$ than that at a higher Mach number.

An extensive experimental investigation has been conducted to study vortex-fin interaction.
on a 76° sharp-edged delta wing with vertical twin-fin configuration by Washburn, Jenkins and Ferman. The vertical tails were placed at nine locations behind the wing. The experimental data showed that the aerodynamic loads are more sensitive to the chordwise tail location than its spanwise location. As the tails were moved toward the vortex core, the buffeting response and excitation were reduced. Although the tail location did not affect the vortex core trajectories, it affected the location of vortex-core breakdown. Moreover, the investigation showed that the presence of a flexible tail can affect the unsteady pressures on the rigid tail on the opposite side of the model. In a recent study by Bean and Lee tests were performed on a rigid 6% scale F/A-18 in a trisonic blowdown wind tunnel over a range of angle of attack and Mach number. The flight data was reduced to a non-dimensional buffet excitation parameter, for each primary mode. It was found that buffeting in the torsional mode occurred at a lower angle of attack and at larger levels compared to the fundamental bending mode.

Kandil, Kandil and Massey presented the first successful computational simulation of the vertical tail buffet using a delta wing-vertical tail configuration. A 76° sharp-edged delta wing has been used along with a single rectangular vertical tail which was placed aft the wing along the plane of geometric symmetry. The tail was allowed to oscillate in bending modes. The flow conditions and wing angle of attack have been selected to produce an unsteady vortex-breakdown flow. Unsteady vortex breakdown of leading-edge vortex cores was captured, and unsteady pressure forces were obtained on the tail. These computational results are in full qualitative agreement with the experimental data of Washburn, Jenkins and Ferman. An alternative simple model for simulation of the buffet problem was used by Kandil and Flanagan and Flanagan. In this model, a configured circular duct was used to produce vortex-breakdown flow through the interaction of a supersonic swirling flow and a shock at the inlet of the duct. Downstream of the vortex-breakdown flow a cantilevered plate was placed. The problem was solved for the quasi-axisymmetric case.

Kandil, Kandil and Massey extended the technique used in Ref. 9 to allow the vertical tail to oscillate in both bending and torsional modes. The total deflections and the frequencies of deflections and loads of the coupled bending-torsion case were found to be one order of magnitude higher than those of the bending case only. Also, it has been shown that the tail oscillations change the vortex breakdown locations and the unsteady aerodynamic loads on the wing and tail.

In this paper, we consider the vortex breakdown flow in the transonic regime, and a 6% scale F/A-18 in a trisonic blowdown wind tunnel over a range of angle of attack and Mach number. The flight data was reduced to a non-dimensional buffet excitation parameter, for each primary mode. It was found that buffeting in the torsional mode occurred at a lower angle of attack and at larger levels compared to the fundamental bending mode.

**FORMULATION**

The formulation of the problem consists of three sets of governing equations along with certain initial and boundary conditions. The first set is the unsteady, compressible, full Navier-Stokes equations. The second set consists of the aeroelastic equations for bending and torsional modes. The third set consists of equations for deforming the grid according to the tail deflections. The literature shows various methods to move the grid. The simplest method uses simple interpolation functions such that the grid points adjacent to the aeroelastic surface move with the surface while the grid points at the computational-region boundary do not move. An alternative simple model for simulation of the buffet problem was used by Kandil and Flanagan and Flanagan. In this model, a configured circular duct was used to produce vortex-breakdown flow through the interaction of a supersonic swirling flow and a shock at the inlet of the duct. Downstream of the vortex-breakdown flow a cantilevered plate was placed. The problem was solved for the quasi-axisymmetric case.

The conservative form of the dimensionless, unsteady, compressible, full Navier-Stokes equations in terms of time-dependent, body-conformed coordinates $\xi^1, \xi^2$ and $\xi^3$ is given by

$$\frac{\partial Q}{\partial t} + \frac{\partial E_m}{\partial \xi^m} - \frac{\partial (E_v)}{\partial \xi^s} = 0; \ m = 1-3, \ s = 1-3 \ (1)$$
where

\[ \xi^m = \xi^m(x_1, x_2, x_3, t) \]  

\[ \mathbf{\tilde{Q}} = \frac{1}{j} [\rho, \rho u_1, \rho u_2, \rho u_3, \rho v, \rho w]^t, \]  

\[ \mathbf{\tilde{Q}_v} \] and (\( \mathbf{\tilde{Q}_v} \)) are the \( \xi^m \)-inviscid flux and \( \xi^v \)-viscous and heat conduction flux, respectively.

Details of these fluxes are given in Ref. 9.

**Aeroelastic Equations:**

The dimensionless, linearized governing equations for the coupled bending and torsional vibrations of a vertical tail that is treated as a cantilevered beam are considered. The tail bending and torsional deflections occur about an elastic axis that is displaced from the inertial axis. These equations for the bending deflection, \( w \), and the twist angle, \( \theta \), are given by

\[
\frac{\partial^2}{\partial z^2} \left[ EI(z) \frac{\partial^2 w}{\partial z^2} (z,t) \right] + m(z) \frac{\partial^2 w}{\partial t^2} (z,t) \\
+ m(z)E(z) \frac{\partial^2 \theta}{\partial z^2} (z,t) = N(z,t) \\
\frac{\partial}{\partial z} \left[ GJ(z) \frac{\partial \theta}{\partial z} \right] - m(z)E(z) \frac{\partial^2 w}{\partial t^2} (z,t) \\
- I_b(z) \frac{\partial^2 \theta}{\partial z^2} (z,t) = -M_1(z,t)
\]  

where \( z \) is the vertical distance from the fixed support along the tail length, \( l_t \), \( EI \) and \( GJ \) the bending and torsional stiffness of the tail section, \( m \) the mass per unit length, \( I_b \) the mass-moment of inertia per unit length about the elastic axis, \( x_\phi \) the distance between the elastic axis and inertia axis, \( N \) the normal force per unit length and \( M_1 \) the twisting moment per unit length. The characteristic parameters for the dimensionless equations are \( c^*, a^\infty, \rho^\infty \) and \( c^*/a^\infty \) for the length, speed, density and time; where \( c^* \) is the delta wing root-chord length, \( a^\infty \) the freestream speed of sound and \( \rho^\infty \) the freestream air density. The geometrical and natural boundary conditions on \( w \) and \( \theta \) are given by

\[
w(0,t) = \frac{\partial w}{\partial z} (0,t) = \frac{\partial^2 w}{\partial z^2} (l_t,t) = 0 \\
\theta(0,t) = \frac{\partial \theta}{\partial z} (l_t,t) = 0
\]  

The solution of Eqs. (4) and (5) are given by

\[
w(z,t) = \sum_{i=1}^{I} \phi_i(z) q_i(t) (8)
\]

\[
\theta(z,t) = \sum_{j=I+1}^{M} \phi_j(z) q_j(t) (9)
\]

where \( \phi_i \) and \( \phi_j \) are comparison functions satisfying the free-vibration modes of bending and torsion, respectively, and \( q_i \) and \( q_j \) are generalized coordinates for bending and torsion, respectively.

In this paper, the number of bending modes, \( I \), is six and the number of torsion modes, \( M - I \), is also six. Substituting Eqs. (8) and (9) into Eqs. (4) and (5) and using the Galerkin method along with integration by parts and the boundary conditions, Eqs (6) and (7), we get the following equation for the generalized coordinates \( q_i \) and \( q_j \) in matrix form:

\[
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\bar{q}_i \\
\bar{q}_j
\end{bmatrix}
+ \begin{bmatrix}
K_{11} & 0 \\
0 & K_{22}
\end{bmatrix}
\begin{bmatrix}
q_i \\
q_j
\end{bmatrix}
= \begin{bmatrix}
\bar{N}_1 \\
\bar{N}_2
\end{bmatrix}
; \quad i = 1, 2, \ldots, I \\
; \quad j = I + 1, \ldots, M
\]

where

\[
M_{11} = \int_0^{l_t} m \phi_i \phi_i dz \\
M_{12} = M_{21} = \int_0^{l_t} m x_\phi \phi_i \phi_j dz \\
M_{22} = \int_0^{l_t} I_b \phi_i \phi_j dz \\
K_{11} = \int_0^{l_t} EI \frac{d^2 \phi_i}{dx^2} \frac{d^2 \phi_i}{dx^2} dz \\
K_{22} = \int_0^{l_t} GJ \frac{d^2 \phi_i}{dx^2} \frac{d^2 \phi_j}{dx^2} dz \\
\bar{N}_1 = \int_0^{l_t} \phi_i N dz \\
\bar{N}_2 = \int_0^{l_t} \phi_j M dz
\]

Similar aeroelastic equations were developed for sonic analysis of wing flutter by Strganac \(^{17,19} \), and Strganac, Mook and Mitchum \(^{18} \). The numerical integration of Eqs. (11-13) is obtained using the trapezoidal method with 125 points to improve the accuracy of integrations. The solution of Eq. (10), for \( q_i; i = 1, 2, \ldots, I \), and \( q_j; j = I + 1, \ldots, M \), is obtained using the Runge-Kutta scheme. Next, \( w \), and \( \theta \) are obtained from Eqs. (8) and (9).
\[ \dot{x}_{II}(t) = A \omega_n \cos \omega_n(t - t_1) - B \omega_n \sin \omega_n(t - t_1) \]  

(7b)

Applying the initial conditions for Region II, Eq. (7) becomes, for \( t_1 < t < t_2 \),

\[ x_{II}(t) = (3 F_C / k - x_0) \cos \omega_n(t - t_1) - F_C / k \]  

(8a)

\[ \dot{x}_{II}(t) = -(3 F_C / k - x_0) \omega_n \sin \omega_n(t - t_1) \]  

(8b)

Next, \( t_2 \) must be determined. \( t_2 \) is the next zero crossing of the velocity function in the region. Again referencing Fig. 4, \( t_2 \) is found to be \( t_2 = t_1 + \frac{\pi}{\omega_n} \). Evaluating Eq. (8) at \( t = t_2 \) yields

\[ x_{II}(t_2) = x_0 - 4 F_C / k \]  

(9a)

\[ \dot{x}_{II}(t_2) = -(3 F_C / k - x_0) \omega_n \sin \omega_n(\frac{\pi}{\omega_n}) = 0 \]  

(9b)

**Determining Coulomb Damping Coefficients**

Using the peak amplitudes of Eqs. (6b) and (9b), the decrease in velocity amplitude from \( t = t_1 \) to \( t = t_2 \) is calculated. The decrease over every half cycle is found to be

\[ (x_0 - F_C / k) \omega_n - (x_0 - 3 F_C / k) \omega_n = (2 F_C / k) \omega_n \]  

(10)

Thus, the velocity amplitude decays by \( (4 F_C / k) \omega_n \) every cycle.

Recalling that \( \omega_n = 2\pi f_n \) where \( f_n \) is the natural frequency in Hz, the decrease in velocity amplitude per cycle, \( \Delta y \), can be written as

\[ \Delta y = \frac{8 F_C \pi}{k} f_n \]

Rearranging, the Coulomb damping force can be determined by

\[ F_C = \frac{\Delta y k}{8 \pi f_n} \]  

(11)

In all probability, more than one cycle will be used to determine the coefficient; thus, the more useful formula is

\[ F_C = \frac{\Delta y k}{8 \pi f_n (\text{no. of cycles})} \]  

(12)

where \( \Delta y \) is redefined as the decrease in amplitude over the number of cycles to be used in determining the Coulomb damping coefficient.

Using Eq. (12), the amount of Coulomb damping in the actuator can be calculated. This is of importance since the friction force can be mathematically removed with nonlinear feedback compensation. In removing this nonlinearity from the system response, it is hoped that a more accurate state-space representation of the testbed will be generated.

**System Identification**

The main goal of this research was to obtain an accurate mathematical representation of the PACOSS DTA from measured frequency response functions. The basic problem posed is one of system identification: Given sampled measurements \( y(k) \) for known inputs \( u(k) \), construct a state-space realization such that the functions \( y(k) \) are accurately reproduced by the state-variable equations.

The Eigensystem Realization Algorithm (ERA) is based upon a singular value decomposition of the block Hankel matrix, which is composed of impulse response data. This modal synthesis technique is capable of identifying the system if there is little or no noise found in the measurements.

Consider the discrete-time linear system:

\[ x(k + 1) = Ax(k) + Bu(k) \]  

(13a)

\[ y(k) = Cx(k) + Du(k) \]  

(13b)

where \( x \) is an \( n \)-dimensional state vector, \( u \) an \( p \)-dimensional input vector, and \( y \) a \( q \)-dimensional output vector. The \( A \), \( B \), \( C \), and \( D \) matrices have dimensions \((n \times n)\), \((n \times p)\), \((q \times n)\), and \((q \times p)\), respectively. The integer \( k \) is the sample indicator. The \( A \) matrix represents the dynamics of the system. For flexible structures, it is a representation of the mass, stiffness, and damping properties.

For the system in Eq. (13) with unit impulse response, the time-domain description is given by the function known as the Markov Parameters.

\[ Y(k) = CA^{k-1}B \]  

(14)

To compute the Markov Parameters, the ERA technique forms a block Hankel matrix. As mentioned before, this matrix is composed of the sampled unit impulse response:

\[ H(k-1) = \begin{bmatrix} Y(k) & \cdots & Y(k + m_{s-1}) \\ \vdots & \ddots & \vdots \\ Y(k + l_{r-1}) & \cdots & Y(k + l_{r-1} + m_{s-1}) \end{bmatrix} \]  

(15)

where \( r \) and \( s \) are arbitrary integers satisfying the inequalities \( r q \geq n \) and \( s p \geq n \), and \( l_i (i = 1, 2, \ldots, r - 1) \) and \( m_j (j = 1, 2, \ldots, s - 1) \) are arbitrary integers. The singular value decomposition of the \( r \times s \) block Hankel matrix is expressed as:

\[ H(0) = PDQ^T \]  

(16)

Using the decomposition of the Hankel matrix, the state-space realization, which is discrete-time and reduced-order, is computed:

\[ A = D^{-1/2}F_n P_n^T H(1)Q_n D_n^{-1/2} \]  

(17a)

In literature the discrete-impulse response function sequence is commonly referred to as the Markov Parameters.
clamped at that edge. The freestream Mach number is 0.85 and the Reynolds number is 3.23 million. The wing angle of attack has been chosen as $20^\circ$ and $28^\circ$. An O-H grid of 65X43X95 grid points in the wrap-around, normal and axial directions, respectively, is used for the solution of the fluid-flow part of the problem. The grid lines in the wake region has been modified to accommodate the tail topology. Figure 1 shows a typical grid and a blow-up of the wing-tail configuration.

**Initial Conditions (Fluid-Flow Problem), $\alpha = 20^\circ$:**

Keeping the tail rigid, the unsteady, compressible, full Navier-Stokes equations are integrated time accurately using the implicit, flux-difference splitting scheme of Roe to a dimensionless time, $t = 10$. Figure 2 shows a three-dimensional view and a top view for the wing-rigid tail configuration. The vortex breakdown of the leading-edge vortex core and the stagnation pressure distribution are shown in the figure. The cross flow beneath the primary vortex reaches supersonic speeds and a ray shock develops beneath the primary vortex. The leading-edge vortex core passes through another transverse shock known as a terminating shock at $x = 0.83$ which causes the vortex core to breakdown at $x = 0.85$.

Figure 3 shows the static pressure contours on the wing surface and symmetry plane. A substantial supersonic pocket which is bounded by the terminating shock and the ray shocks (shocks beneath the primary vortex cores) is observed on the wing plane. Figure 4 shows the Mach contours and streamlines on a vertical ray plane (ray D) which passes through the vortex breakdown. The streamlines conclusively show a two-bubble cell vortex breakdown. The Mach contours show that the front surface of the vortex breakdown bubbles is enclosed by a hemispherical shape-like shock surface. Figure 5 shows the static-pressure variation along ray lines from the wing vertex. These curves show the spanwise locations of several points on the foot-print line of the terminating shock. The terminating shock is clearly seen to run in the spanwise direction from the plane of symmetry to the wing leading edge.

The solution at the present time step is taken as the initial conditions for the next case of the aeroelastic tail response.

**Uncoupled Bending-Torsion Tail Response, $\alpha = 20^\circ$:**

The tail is treated as a swept back beam with thickness $d = 0.005$. The tail material dimensionless moduli of elasticity and rigidity, $E$ and $G$ are $1.8X10^5$ and $0.692X10^5$, respectively. The mass per unit length of the tail varies linearly from the tail root, $m_r = 0.033$, to the tail tip, $m_t = 0.0076$ and the mass-moment of inertia per unit length varies linearly from the tail root, $I_{gr} = 1.75X10^{-4}$, to the tail tip, $I_{gt} = 2.1X10^{-6}$. For the coupled bending-torsion case, the elastic axis is assumed to exist upstream the inertia axis with a distance of $x_\theta = -0.02$. For the uncoupled bending-torsion case, $x_\theta = 0.0$.

Figures 6-8 show the results of the uncoupled bending-torsion responses of the tail. Figure 6 shows four pairs of responses. The first pair is for the variation of the bending deflection, $w$, and torsional deflection, $\theta$, along the tail height $z$ every 2000 time step. The bending and torsion responses are mainly of the first-mode shape type. The second pair of responses show the variation of the normal force and twisting moment along the tail height $z$ every 2000 time steps. The third and fourth pairs show the bending deflection, normal force, torsional deflection and twisting moment variation at the tail tip and its midpoint versus the number of time steps ($it = 20,000$ or $t = 20$ starting from the initial condition). It is observed that the frequency of the normal force and the twisting moment are almost the same as that of the bending deflection and the torsional deflection, respectively. Figure 7 shows the combined response, $W_{net}$, of the bending and torsional deflections along the tail height every 2000 time steps. Figure 8 shows a three-dimensional view and a top view of the wing-deformed tail configuration at $it = 20,000$. Comparing this figure with Fig. 2 (Initial condition with rigid tail), the terminating shock moves upstream to $x = 0.5$ and becomes weaker and smeared. The vortex breakdown occurs immediately after the terminating shock. Another shock is observed downstream of the original terminating shock and is accompanied by another breakdown. The breakdown flow is slightly asymmetric. This conclusively shows the substantial upstream aerodynamic effects of the tail bending and torsional deflections.
Initial Conditions (Fluid-Flow Problem),\[ \alpha = 28^\circ: \]
Keeping the tail rigid, the angle of attack is increased to 28°. The other flow conditions are kept the same as those of \( \alpha = 20^\circ \). Figure 9 shows a three-dimensional view and a top view of the vortex breakdown of the leading-edge vortex cores and the stagnation pressure contours. The vortex breakdown flow moves upstream covering almost all of the wing planform. Figure 10 shows the Mach contours on a constant K plane near the wing surface and on the plane of symmetry. The supersonic pocket on the upper wing surface expanded in the spanwise direction to cover all of the wing planform, and part of the transverse terminating shock moved downstream ahead of the vertical tail location. Figure 11 shows the variation of static pressures along ray planes originating from the wing planform vertex. It is observed that several transverse shocks exist; one near the wing vertex at \( x/c = 0.2 \), a second one at \( x/c = 0.9 \) and a third one ahead of the tail location at \( x/c = 1.06 \).

Uncoupled Bending-Torsion Tail Response, \( \alpha = 28^\circ: \)
Figures 12-15 show the results of this case. Figure 12 shows the same sequence of results as those of Fig. 6. Comparing the results of Fig. 12 with those of Fig. 6, it is observed that the bending and torsional deflections of Fig. 12 are 4-5 times as those of Fig. 6. Moreover, the frequencies of the bending and torsional deflections and loads of Fig. 12 are lower than those of Fig. 6. The aerodynamic damping of the case of Fig. 6 is higher than that of the case of Fig. 12. The net deflection of Fig. 13 is 4 times higher than that of Fig. 7. Figure 14 shows a three-dimensional view of the vortex breakdown of the leading-edge vortex cores and the stagnation pressure at \( it = 20,000 \). It is observed that the leading critical points of the vortex breakdown near the wing vertex are asymmetric. Figure 15 shows the Mach contours on a constant K plane near the wing surface and on the plane of symmetry. The Mach contours on the wing surface show strong asymmetry with one side having a subsonic flow and the other side having a supersonic flow. It is observed that a shock near the plane of symmetry and originating from the wing vertex exists.

CONCLUDING REMARKS
The tail buffet problem due to the unsteady aerodynamic loads induced by the vortex-breakdown flow of the wing leading-edge vortices has been simulated computationally and efficiently using a delta wing-swept back vertical tail configuration. The wing aspect ratio and flow conditions (transonic regime) have been carefully selected in order to produce unsteady vortex-breakdown flow. The solution has demonstrated the development of the tail buffet due to the unsteady loads produced by the transonic vortex-breakdown flow. The problem is a multidisciplinary problem which requires three sets of equations to obtain its solution.

In the present paper, the CFD solver is the implicit, upwind, Roe flux-difference splitting scheme.

The focus of this paper is to study the buffet response in transonic flow at different angles of attack. It is conclusively found that the tail oscillations have a substantial upstream effect on the vortex breakdown of the leading-edge vortex cores, although a supersonic pocket exists on the wing upper surface. By increasing the angle of attack from 20° to 28°, the vortex breakdown flow becomes stronger and the corresponding unsteady normal forces and torsional moments on the tail become larger resulting into substantially higher deflections with lower frequencies. Unlike the results obtained in Ref. 13 for subsonic vortex breakdown flows and a delta wing-rectangular vertical tail configuration, the aerodynamic loads and the deflections in the present case never reached periodic response and their loads were one order of magnitude lower than those of Ref. 13. These results are in a qualitative agreement with the conclusion reached by Cole, Moss and Doggett of Ref. 6; That the buffet deflections become larger as the Mach number is decreased.

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Fig. 1  An O-II grid of 65X43X95 grid points in the wrap-around, normal and axial directions and a blow-up of the wing-tail configuration.
Fig. 2 Three-dimensional view and a top view of the wing-tail configuration for the initial conditions, $\alpha = 20^\circ$, $\Delta t = 0.001$, $it = 10,000$. 
Fig. 3  Pressure contours on the wing surface and plane of symmetry for the initial conditions, $\alpha = 20^\circ$, $\Delta t = 0.001$, $it = 10,000$.

Fig. 4  Total Mach contours and streamlines on a ray plane passing through the vortex-breakdown, $\alpha = 20^\circ$, $\Delta t = 0.001$, $it = 10,000$.

Fig. 5  Ray planes on the wing planform and surface pressure variation along these planes, $\alpha = 20^\circ$, $\Delta t = 0.001$, $it = 10,000$. 
Figure 6: Deflection and load response for an uncoupled bending-torsion case.

Deflection and Load Response for an Uncoupled Bending-Torsion Case.
Fig. 7  Net deflection of the tail leading edge for the uncoupled bending-torsion case, $\alpha = 20^\circ$, $\Delta t = 0.001$, $it = 10,000 - 30,000$.

Fig. 8  Three-dimensional view and a top view of the wing-tail configuration for the uncoupled bending-torsion case, $\alpha = 20^\circ$, $\Delta t = 0.001$, $it = 20,000$. 
Fig. 9 Three-dimensional view and a top view of the wing-tail configuration for the initial conditions, $\alpha = 28^\circ$, $\Delta t = 0.001$, $it = 10,000$. 

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Fig. 10  Mach contours on a constant K plane near the wing surface and on the plane of symmetry, $\alpha = 28^\circ$, $\Delta t = 0.001$, $it = 10,000$.

Fig. 11  Ray planes on the wing planform and surface pressure variation along these planes, $\alpha = 28^\circ$, $\Delta t = 0.001$, $it = 10,000$. 

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Deflection and load responses for an uncoupled bending-torsion case,
\( \alpha = 28^\circ, \Delta t = 0.001, n = 10,000 - 30,000. \)
Fig. 13  Net deflection of the tail leading edge for the uncoupled bending-torsion case, $\alpha = 28^\circ$, $\Delta t = 0.001$, $it = 10,000 - 30,000$.

Fig. 14  Three-dimensional view and a top view of the wing-tail configuration for the uncoupled bending-torsion case, $\alpha = 28^\circ$, $\Delta t = 0.001$, $it = 20,000$.

Fig. 15  Mach contours on a constant K plane near the wing surface and on the plane of symmetry, $\alpha = 28^\circ$, $\Delta t = 0.001$, $it = 20,000$. 